

INTEGRAL INEQUALITIES FOR SECOND-ORDER LINEAR OSCILLATION

MAN KAM KWONG

Abstract. We present several results related to the classical Lyapunov inequality for the oscillation of second-order linear equations. The first is an improved Lyapunov inequality given in terms of the downswing of the functions $\int_a^t (t-a)q(t) dt$ and $\int_t^b (b-t)q(t) dt$, extending earlier results of Kwong and Harris and Kong. Nonoscillation criteria are derived as corollaries. A Lyapunov-type inequality for two consecutive zeros of the derivative of a solution is then established and a nonoscillation criterion given as a corollary. An oscillation criterion for positive $q(t)$ is also proved. It extends the known condition $\int t^\gamma q(t) dt = \infty$, $\gamma \in [0, 1)$.

Mathematics subject classification (1991): 34C10, 26D10.

Key words and phrases: Lyapunov inequality, oscillation, second-order linear differential equation.

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