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AN OPERATOR INEQUALITY WHICH IMPLIES PARANORMALITY

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Abstract. Let T be a bounded linear operator on a Hilbert space. Among other things, it is shown that (1) if $|T^2| \geqslant |T|^2$, then T is paranormal, (2) if T is w-hyponormal, then $|T^2| \geqslant |T|^2$ and $|T^{*2}| \leqslant |T^*|^2$, and (3) if T and T^* are w-hyponormal, and either $\ker T \subseteq \ker T^*$ or $\ker T^* \subseteq \ker T$, then T is normal.

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