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ON A NEW EQUIVALENCE OF COEFFICIENT CONDITIONS AND APPLICATIONS

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Abstract. We show that if the sequence $\{\kappa_n\}$ is quasi geometrically increasing, then a so-called block-condition

$$\sum_{m=0}^{\infty} \kappa_m \left(\sum_{n=\nu_m+1}^{\nu_{m+1}} |c_n|^q \right)^{p/q} < \infty, \quad 0 < p < q,$$

for every $\{\nu_m\}$ is equivalent to the following two conditions

$$\sum_{n=1}^{\infty} |c_n|^q \mu_n < \infty \quad \text{and} \quad \sum_{n=1}^{\infty} \kappa_n \left(\frac{\kappa_n}{\mu_{\nu_n+1}} \right)^{\frac{p}{q-p}} < \infty,$$

where $\{\mu_n\}$ is a nondecreasing sequence.

Applications to absolute $|C,\alpha|$-summability of general orthogonal series are also presented.

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