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INEQUALITIES FOR SOME COEFFICIENTS OF UNIVALENT FUNCTIONS

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Abstract. Let $\mathscr{S}$ be the usual class of normalized analytic and univalent functions in the open unit disk. We write

$$\log \frac{f(z)}{z} = 2\sum_{n=1}^{\infty} \gamma_n z^n \quad (f \in \mathscr{S}).$$

The well-known de Branges' theorem shows that

$$I_n = \sum_{k=1}^{n} (n-k+1)\left(k|\gamma_k|^2 - \frac{1}{k}\right) \leqslant 0 \quad (n \in \mathbb{N} := \{1,2,3,\cdots\};\ f \in \mathscr{S}).$$

In this paper we use the properties of I_n to obtain some coefficient inequalities for univalent functions. The results obtained here extend and unify several known results.

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