# SHARP INEQUALITIES CONNECTED TO THE
# HOMOGENIZED $p$–POISSON EQUATION

### DAG LUKKASSEN

*Abstract.* In this paper we study inequalities for the effective energy density associated with the homogenized $p$-Poisson equation. We prove that the inequalities are sharpest possible and we even find all cases of equality. Our results implies uniqueness of rank 1 laminates within the class of multi-phase structures.

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