

A STRENGTHENED CAUCHY—SCHWARZ INEQUALITY FOR BIORTHOGONAL WAVELETS

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Abstract. A strengthened Cauchy–Schwarz inequality for spaces of biorthogonal wavelets defined on the real line and on the interval is proved. The strengthened Cauchy–Schwarz inequality is a fundamental tool in the analysis of the multilevel methods and, in particular, plays an important role in the a posteriori error estimates for hierarchical methods.

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