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SOME APPLICATIONS OF TANAHASHI'S RESULT ON THE BEST POSSIBILITY OF FURUTA INEQUALITY

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Abstract. We shall give some applications of Tanahashi's result which states the best possibility of Furuta inequality. Firstly, we shall discuss the best possibility of a well-known characterization of chaotic order: $\log A \geqslant \log B$ *if and only if* $A^r \geqslant (A^{\frac{r}{2}} B^p A^{\frac{r}{2}})^{\frac{r}{p+r}}$ *holds for all* $p \geqslant 0$ *and* $r \geqslant 0$. Secondly, we shall discuss the best possibility of p-hyponormality of generalized Aluthge transformation $\tilde{T}_{s,t} = |T|^s U |T|^t$ for p-hyponormal or log-hyponormal operator T whose polar decomposition is $T = U|T|$.

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REFERENCES

[1] A. ALUTHGE, *On p-hyponormal operators for $0 < p < 1$*, Integral Equations Operators Theory **13** (1990), 307–315.
[2] T. ANDO, *On some operator inequalities*, Math. Ann. **279** (1987), 157–159.
[3] M. FUJII, *Furuta's inequality and its mean theoretic approach*, J. Operator Theory **23** (1990), 67–72.
[4] M. FUJII, T. FURUTA AND E. KAMEI, *Furuta's inequality and its application to Ando's theorem*, Linear Algebra Appl. **179** (1993), 161–169.
[5] M. FUJII, J. F. JIANG AND E. KAMEI, *Characterization of chaotic order and its application to Furuta inequality*, Proc. Amer. Math. Soc. **125** (1997), 3655–3658.
[6] M. FUJII, J. F. JIANG AND E. KAMEI, *A characterization of orders defined by $A^\delta \geqslant B^\delta$ via Furuta inequality*, Math. Japon. **45** (1997), 519–525.
[7] M. FUJII, J. F. JIANG, E. KAMEI AND K. TANAHASHI, *A characterization of chaotic order and a problem*, J. Inequal. Appl. **2** (1998), 149–156.
[8] T. FURUTA, *$A \geqslant B \geqslant 0$ assures $(B^r A^p B^r)^{1/q} \geqslant B^{(p+2r)/q}$ for $r \geqslant 0$, $p \geqslant 0$, $q \geqslant 1$ with $(1+2r)q \geqslant p+2r$*, Proc. Amer. Math. Soc. **101** (1987), 85–88.
[9] T. FURUTA, *An elementary proof of an order preserving inequality*, Proc. Japan Acad. Ser. A Math. Sci. **65** (1989), 126.
[10] T. FURUTA, *Applications of order preserving operator inequalities*, Oper. Theory Adv. Appl. **59** (1992), 180–190.
[11] T. FURUTA AND M. YANAGIDA, *Further extensions of Aluthge transformation on p-hyponormal operators*, Integral Equations Operator Theory **29** (1997), 122–125.
[12] T. HURUYA, *A note on p-hyponormal operators*, Proc. Amer. Math. Soc. **125** (1997), 3617–3624.
[13] E. KAMEI, *A satellite to Furuta's inequality*, Math. Japon. **33** (1988), 883–886.
[14] K. TANAHASHI, *Best possibility of the Furuta inequality*, Proc. Amer. Math. Soc. **124** (1996), 141–146.
[15] K. TANAHASHI, *The best possibility for the grand Furuta inequality* (Japanese), Recent topics in operator theory concerning the structure of operators (Kyoto, 1996), RIMS Kōkyūroku **979** (1997), 1–14.
[16] K. TANAHASHI, *On log-hyponormal operators*, preprint.
[17] T. YOSHINO, *The p-hyponormality of the Aluthge transform*, Interdiscip. Inform. Sci. **3** (1997), 91–93.

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