Mathematical
Inequalities
& Applications
Volume 2, Number 2 (1999), 307–316

# COMPLEMENTARY HALFSPACES AND TRIGONOMETRIC CEVA–BROCARD INEQUALITIES FOR POLYGONS

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*Abstract.* The product of ratios that equals 1 in Ceva's Theorem is analyzed in the case of non-concurrent Cevians, for triangles as well as arbitrary convex polygons. A general lemma on complementary systems of inequalities is proved, and used to classify the possible cases of non-concurrent Cevians. In the concurrent case, particular consideration is given to the Brocard configuration defined by equal angles between Cevians and polygon sides.

*Mathematics subject classification* (1991): 51M16, 51M20, 51M04, 51M15.
*Key words and phrases*: Triangle geometry, Brocard angle, Brocard points, Ceva theorem, convex polygons, half-spaces, inequalities.

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