

SEVERAL APPROXIMATIONS OF $\pi(x)$

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Abstract. In this paper several new inequalities on the function $\pi(x)$ (numbers of primes not exceeding x) are presented. In the proofs, essentially the well-known results of Rosser and Schoenfeld are used.

Legendre conjectured that $x/(\log x - A)$ (with $A = 1.08366\dots$) is a good approximation for $\pi(x)$. We prove that, for $x > 10^6$, the function considered by Legendre is actually an upper bound.

L. Locker-Ernst affirms that $\frac{n}{h(n)}$, with $h(n) = \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$ is very close to $\pi(n)$. We precize the above statement by proving that, for $n \geq 1429$, $\frac{n}{h(n)}$ is actually a lower bound for $\pi(n)$.

Mathematics subject classification (1991): 11A25, 11N05.

Key words and phrases: Inequalities for arithmetic functions, bounds for integrals of convex functions, Legendre conjecture.

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