

THE MAPPING $\Psi_{x,y}^p$ IN NORMED LINEAR SPACES AND ITS APPLICATIONS IN THE THEORY OF INEQUALITIES

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Abstract. In this paper we introduce the mapping $\Psi_{x,y}^p(t) = (x, x + ty)_p \|x + ty\|^{-1}$, which is derived from the lower and upper semi-inner product $(\cdot, \cdot)_i$ and $(\cdot, \cdot)_s$, and study its properties of monotonicity, boundedness and convexity. We give applications to height functions and to inequalities in analysis, including a refinement of the Schwarz inequality.

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