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RIESZ'S FUNCTIONS AND CARLESON INEQUALITIES

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Abstract. Let μ be a finite positive Borel measure on the open unit disc D and H a set of all analytic functions on D. For each a in D, put

$$r(\mu, a) = \sup |f(a)|^2$$

where $f \in H$ and $\int_D |f|^2 d\mu \leqslant 1$. Unless the support set of μ is a finite set, $\int_D r(\mu, a) d\mu(a) = \infty$. However

$$\sup_{z \in D} \int_{D_t(z)} r(\mu, a) d\mu(a) < \infty$$

may happen where $D_t(z)$ denotes the Bergman disc in D. We study when this is possible. When ν is a discrete measure such that $d\nu = \sum_{a \in A} s(\mu, a)\delta_a$,

$$\sup_{z \in D} \int_{D_t(z)} r(\mu, a) d\nu(a) = \sup_{z \in D} \sum_{a \in A \cap D_t(z)} 1.$$

Under some condition on μ, we show that $\sup_{z \in D} \int_{D_t(z)} r(\mu, a) d\nu(a) < \infty$ for a finite positive Borel measure ν on D if and only if (ν, μ)-Carleson inequality is valid.

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