RIESZ’S FUNCTIONS AND CARLESON INEQUALITIES

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Abstract. Let \( \mu \) be a finite positive Borel measure on the open unit disc \( D \) and \( H \) a set of all analytic functions on \( D \). For each \( a \) in \( D \), put

\[
r(\mu, a) = \sup |f(a)|^2
\]

where \( f \in H \) and \( \int_D |f|^2 \, d\mu \leq 1 \). Unless the support set of \( \mu \) is a finite set, \( \int_D r(\mu, a) \, d\mu(a) = \infty \). However

\[
\sup_{z \in D} \int_{D(z)} r(\mu, a) \, d\mu(a) < \infty
\]

may happen where \( D(z) \) denotes the Bergman disc in \( D \). We study when this is possible. When \( \nu \) is a discrete measure such that \( d\nu = \sum a \in A \, s(\mu, a) \, \delta_a \),

\[
\sup_{z \in D} \int_{D(z)} r(\mu, a) \, d\nu(a) = \sup_{z \in D} \sum_{a \in A \cap D(z)} 1.
\]

Under some condition on \( \mu \), we show that \( \sup_{z \in D(z)} r(\mu, a) \, d\nu(a) < \infty \) for a finite positive Borel measure \( \nu \) on \( D \) if and only if \( (\nu, \mu) \)-Carleson inequality is valid.

Key words and phrases: Bergman space, weight, Riesz’s function, Carleson inequality, interpolation sequence.

REFERENCES