ON GENERALIZED LORENTZ—ZYGMUND SPACES

B. OPIC AND L. PICK

Abstract. We study generalized Lorentz–Zygmund spaces with broken logarithmic functions. We derive necessary and sufficient conditions for embeddings between them. We give a complete characterization of their associate spaces. We establish necessary and sufficient conditions for a generalized Lorentz–Zygmund space to be a Banach function space and to have absolutely continuous (quasi-)norm. We describe completely relations between these spaces and Orlicz spaces.

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