

## SEVERAL PROPERTIES ON CLASS $A$ INCLUDING $p$ -HYPONORMAL AND LOG-HYPONORMAL OPERATORS

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*Abstract.* In this paper, we shall show that Yamazaki's result in [23] remains valid for class  $A$  introduced in [13] as a class of operators including  $p$ -hyponormal and log-hyponormal operators. Moreover we shall show several properties on class  $A$  which correspond to the properties on paranormal in [8][12][18] and [19].

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