

## UNIFYING APPROACH TO THE STUDY OF $p$ -HYPONORMAL OPERATORS VIA FURUTA INEQUALITY

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*Abstract.* In this paper we propose a unifying approach to the study of the operators  $|T|^g U |T|^h$  and  $|T^*|^g U |T^*|^h$  being  $p$ -hyponormal. This approach is based on the direct use of combined Furuta inequalities, and, consequently, characterizations of a normal operator are presented.

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