Mathematical
Inequalities
& Applications
Volume 2, Number 4 (1999), 585–597

EXISTENCE THEOREMS OF GENERALIZED QUASI–VARIATIONAL INEQUALITIES WITH UPPER HEMI–CONTINUOUS AND DEMI OPERATORS ON NON–COMPACT SETS

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Abstract. Suppose that E is a topological vector space and X is a non-empty subset of E. Let $S : X \to 2^X$ and $T : X \to 2^{E^*}$ be two maps. Then the generalized quasi-variational inequality problem (GQVI) is to find a point $\hat{y} \in S(\hat{y})$ and a point $\hat{w} \in T(\hat{y})$ such that $Re\langle \hat{w}, \hat{y} - x\rangle \leqslant 0$ for all $x \in S(\hat{y})$. We shall use Chowdhury and Tan's generalized version [4] of Ky Fan's minimax inequality [7] as a tool to obtain some general theorems on solutions of the GQVI in locally convex Hausdorff topological vector spaces. We obtain the existence theorems of GQVI on paracompact sets X where the set-valued operators T are demi operators [3] and are upper hemi-continuous [5] along line segments in X to the weak * -topology on E^*.

Mathematics subject classification (1991): 47H04, 47H05, 47H09, 47H10, 49J35, 49J40, 54C60.
Key words and phrases: Generalized quasi-variational inequality, h-demi operators, demi operators, lower semi-continuous, upper semi-continuous and upper hemi-continuous operators.

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Paper MIA-02-50