

REMARKS, QUESTIONS AND CONJECTURES ON LANDAU-KOLMOGOROV-TYPE INEQUALITIES

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Abstract. Results on Landau-Kolmogorov-type inequalities are surveyed with some new results given and some open questions raised. A multivariate analogue and analogues using different operators than the derivative are particularly emphasized. Equivalence between the periodic and nonperiodic case is shown. Failure of Kolmogorov's inequality for L_p spaces when $0 < p < 1$ is demonstrated.

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