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SHARP INEQUALITIES FOR SOLUTIONS OF MULTIPOINT BOUNDARY VALUE PROBLEMS

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Abstract. This paper considers the following continuous and discrete multipoint boundary value problems: $x^{(n)}(t) \geqslant 0,\ 0 \leqslant t \leqslant 1,\ x^{(j)}(t_i) = 0$ and $\Delta^n y(k) \geqslant 0,\ k = 0, \cdots, m,\ \Delta^j y(k_i) = 0$, where $j = 0, \cdots, n_i - 1,\ i = 1, \cdots, r,\ \sum_{i=1}^{r} n_i = n,\ 0 = t_1 < t_2 < \cdots < t_r = 1$, and $0 = k_1 < k_1 + n_1 < k_2 < k_2 + n_2 < \cdots < k_r \leqslant k_r + n_r - 1 = m + n$. We offer lower bounds for solutions of these boundary value problems in terms of $\sup_{0 \leqslant t \leqslant 1} |x(t)|$ and $\max_{k \in \{0, \cdots, m+n\}} |y(k)|$. These bounds further lead to inequalities for related Green's functions which are very useful in the study of positive solutions of boundary value problems.

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