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AN EXTENSION OF SPECHT'S THEOREM VIA KANTOROVICH INEQUALITY AND RELATED RESULTS

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Abstract. In this paper, we shall show the following result.

"If $MI \geqslant A \geqslant mI > 0$ with $M > m > 0$, then

$$K_+(m^r, M^r, \tfrac{p}{r})^{\frac{1}{p}} (A^r x, x)^{\frac{1}{r}} \geqslant (A^p x, x)^{\frac{1}{p}}$$

for $p > r > 0$, where

$$K_+(m, M, p) = \frac{(p-1)^{p-1}}{p^p} \frac{(M^p - m^p)^p}{(M-m)(mM^p - Mm^p)^{p-1}}."$$

This result is an extension of Specht's theorem [6] as a converse of the arithmetic-geometric mean inequality.

"If $x_1, x_2, \cdots, x_n \in [m, M]$ with $M > m > 0$, then

$$M_h \sqrt[n]{x_1 x_2 \cdots x_n} \geqslant \frac{x_1 + x_2 + \cdots + x_n}{n}$$

where $h = \frac{M}{m} > 1$ and $M_h = \dfrac{h^{\frac{1}{h-1}}}{e \log h^{\frac{1}{h-1}}}$."

Secondly, we shall show an application for operator inequalities, that is, *"if $A \geqslant B \geqslant 0$ satisfying $MI \geqslant B \geqslant mI > 0$ with $M > m > 0$, then*

$$A^p - B^p \geqslant \frac{-(mM^p - Mm^p)}{M-m} \left\{ K_+(m, M, p)^{\frac{1}{p-1}} - 1 \right\}$$

for $p > 1$."

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REFERENCES

[1] J. I. FUJII, T. FURUTA, T. YAMAZAKI AND M. YANAGIDA, *Simplified proof of characterization of chaotic order via Specht's ratio*, Scientiae Mathematicae, **2** (1999), 63–64.
[2] J. I. FUJII, S. IZUMINO AND Y. SEO, *Determinant for positive operators and Speht's theorem*, Scientiae Mathematicae, **1** (1998), 307–310.
[3] J. I. FUJII AND Y. SEO, *Determinant for positive operators*, Scientiae Mathematicae, **1** (1998), 153–156.
[4] T. FURUTA, *Operator inequalities associated with Hölder-McCarthy and Kantorovich inequalities*, J. Inequal. Appl., **2** (1998), 137–148.
[5] J. MIĆIĆ, Y. SEO, S-E. TAKAHASHI AND M. TOMINAGA, *Inequalities of Furuta and Mond-Pečarić*, Math. Ineq. Appl., **2** (1999), 83–111.
[6] W. SPECHT, *Zur Theorie der elementaren Mittel*, Math. Z., **74** (1960), 91–98.
[7] T. YAMAZAKI AND M. YANAGIDA, *Characterizations of chaotic order associated with Kantorovich inequality*, Scientiae Mathematicae, **2** (1999), 37–50.

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