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ON POWERS OF CLASS A(k) OPERATORS INCLUDING p–HYPONORMAL AND LOG–HYPONORMAL OPERATORS

TAKEAKI YAMAZAKI

Abstract. In [11], we introduced class A(k) as a class of operators including p-hyponormal and log-hyponormal operators. In this paper, we shall show that "if T is an invertible class A(k) operator for $k \in (0,1]$, then T^n is a class A($\frac{k}{n}$) operator for all positive integer n." Moreover, we shall show a similar result on powers of class AI(s,t) operators which were introduced in [7] as extensions of class A(k) operators, that is, "if T is a class AI(s,t) operator for $s,t \in (0,1]$, then T^n is a class AI($\frac{s}{n},\frac{t}{n}$) operator for all positive integer n."

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