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THE HAUSDORFF AND THE QUASI HAUSDORFF OPERATORS ON THE SPACES $L^p, 1 \leqslant p < \infty$

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Abstract. The operators indicated in the title are defined by means of Lebesgue-Stieltjes integrals of real- (or complex-) valued funtions with respect to σ-finite positive (or signed, or complex) measures μ defined on the Borel measurable subsets of $\mathbf{R}_+$ (or $\mathbf{R}$). We give simple sufficient conditions in terms of μ in order that these operators be bounded on the Lebesgue space $L^p(\mathbf{R}_+)$ (or $L^p(\mathbf{R})$) for some $1 \leqslant p < \infty$. These sufficient conditions are exact even in the wellknown special cases of the Cesàro and Copson operators. We also prove that the Hausdorff and the quasi Hausdorff operators are adjoint of one another, under an appropriate condition in terms of μ. On closing, we reveal an interrelation among these operators and the Fourier transform of a function in $L^1(\mathbf{R})$.

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