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INEQUALITIES FOR THE DERIVATIVES

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Abstract. The following question is studied and answered:

Is it possible to stably approximate f' if one knows:

1) $f_\delta \in L^\infty(\mathbf{R})$ such that $\|f - f_\delta\| < \delta$,

and

2) $f \in C^\infty(\mathbf{R})$, $\|f\| + \|f'\| \leqslant c$?

Here $\|f\| := \sup_{x\in\mathbf{R}} |f(x)|$ and $c > 0$ is a given constant. By a stable approximation one means $\|L_\delta f_\delta - f'\| \leqslant \eta(\delta) \to 0$ as $\delta \to 0$. By $L_\delta f_\delta$ one denotes an estimate of f'. The basic result of this paper is the inequality for $\|L_\delta f_\delta - f'\|$, a proof of the impossibility to approximate stably f' given the above data 1) and 2), and a derivation of the inequality $\eta(\delta) \leqslant c\delta^{\frac{a}{1+a}}$ if 2) is replaced by $\|f\|_{1+a} \leqslant m_{1+a}$, $0 < a \leqslant 1$. An explicit formula for the estimate $L_\delta f_\delta$ is given.

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REFERENCES

[1] G. HARDY, J. LITTLEWOOD, G. POLYA, *Inequalities*, Cambridge Univ. Press, London, 1951.
[2] A. KOLMOGOROV, *On inequalities between derivatives*, Uchen. Zapiski Moscow Univ. Math., **30**, (1939), 3–16.
[3] E. LANDAU, *Einige Ungleichungen für zweimal differentierbare Functionen*, Proc. Lond. Math. Soc., **2**, N13, (1913), 43–49.
[4] T. MILLER, A. G. RAMM, *Estimates of the derivatives of random functions II.*, J. Math. Anal. Appl., **110**, (1985), 429–435.
[5] A. G. RAMM, *Random fields estimation theory*, Longman Scientific and Wiley, New York, 1990.
[6] ______, *On numerical differentiation*, Izvestija vuzov, Mathem., **11**, (1968), 131–135.
[7] ______, *Stable solutions of some ill-posed problems*, Math. Meth. in appl. Sci., **3**, (1981), 336–363.
[8] ______, *Estimates of the derivatives of random functions*, J. Math. Anal. Appl., **102**, (1984), 244–250.
[9] ______, *On simultaneous approximation of a function and its derivative by interpolation polynomials*, Bull. Lond. Math. Soc., **9**, (1977), 283–288.
[10] ______, *Simplified optimal differentiators*, Radiotech.i Electron., **17**, (1972), 1325–1328.
[11] A. G. RAMM, A. B. SMIRNOVA, *On stable numerical differentiation*, (to appear).

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