

## CONVEXITY ACCORDING TO THE GEOMETRIC MEAN

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*Abstract.* We develop a parallel theory to the classical theory of convex functions, based on a change of variable formula, by replacing the arithmetic mean by the geometric one. It is shown that many interesting functions such as  $\exp$ ,  $\sinh$ ,  $\cosh$ ,  $\sec$ ,  $\csc$ ,  $\arcsin$ ,  $\Gamma$  etc illustrate the multiplicative version of convexity when restricted to appropriate subintervals of  $(0, \infty)$ . As a consequence, we are not only able to improve on a number of classical elementary inequalities but also to discover new ones.

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