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WEIGHTED INEQUALITY FOR SOME CLASSICAL INTEGRAL OPERATORS: $0 < p < 1$

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Abstract. Suppose $0 < p < 1$ and $0 < q < \infty$. In this note, we prove that the weighted (p, q) inequality

$$\left(\int_0^\infty (Tf(x))^q w(x)dx\right)^{1/q} \leqslant C\left(\int_0^\infty (f(x))^p v(x)dx\right)^{1/p}$$

has no nontrivial solution if Tf is a Hardy type operator, the Hardy-Littlewood maximal operator or an one-sided maximal operator.

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