

FURTHER CHARACTERIZATIONS OF CHAOTIC ORDER VIA SPECHT'S RATIO

TAKEAKI YAMAZAKI

Abstract. As a characterization of chaotic order, we showed “If $MI \geq B \geq mI > 0$, then $\log A \geq \log B$ is equivalent to

$$M_h(p)A^p \geq B^p$$

for all $p > 0$, where $h = \frac{M}{m} > 1$ and $M_h(p) = \frac{h^{\frac{p}{h^p-1}}}{e \log h^{\frac{p}{h^p-1}}}$ ” in [11].

In this paper, we shall show the following characterization of chaotic order as a parallel result to the result mentioned above:

“If $MI \geq B \geq mI > 0$, then $\log A \geq \log B$ is equivalent to

$$A^p + L(m^p, M^p) \log M_h(p)I \geq B^p$$

for all $p > 0$, where $L(m, M) = \frac{M-m}{\log M - \log m}$.” And we shall discuss the relations among this result and some related results.

Mathematics subject classification (1991): 47A63.

Key words and phrases: Positive operator, Specht's ratio, chaotic order.

REFERENCES

- [1] T. ANDO, *On some operator inequalities*, Math. Ann., **279** (1987), 157–159.
- [2] J. I. FUJII, T. FURUTA, T. YAMAZAKI AND M. YANAGIDA, *Simplified proof of characterization of chaotic order via Specht's ratio*, Scientiae Mathematicae, **2** (1999), 63–64.
- [3] J. I. FUJII, S. IZUMINO AND Y. SEO, *Determinant for positive operators and Specht's theorem*, Scientiae Mathematicae, **1** (1998), 307–310.
- [4] J. I. FUJII AND Y. SEO, *Determinant for positive operators*, Scientiae Mathematicae, **1** (1998), 153–156.
- [5] M. FUJII, J. F. JIANG AND E. KAMEI, *Characterization of chaotic order and its application to Furuta inequality*, Proc. Amer. Math. Soc., **125** (1997), 3655–3658.
- [6] T. FURUTA, *Applications of order preserving operator inequalities*, Oper. Theory Adv. Appl., **59** (1992), 180–190.
- [7] T. FURUTA, *Operator inequalities associated with Hölder-McCarthy and Kantorovich inequalities*, J. Inequal. Appl., **2** (1998), 137–148.
- [8] W. SPECHT, *Zur Theorie der elementaren Mittel*, Math. Z., **74** (1960), 91–98.
- [9] M. UCHIYAMA, *Some exponential operator inequalities*, Math. Inequal. Appl., **2** (1999), 469–471.
- [10] T. YAMAZAKI, *An extension of Specht's theorem via Kantorovich inequality and related results*, to appear in Math. Inequal. Appl.
- [11] T. YAMAZAKI AND M. YANAGIDA, *Characterizations of chaotic order associated with Kantorovich inequality*, Scientiae Mathematicae, **2** (1999), 37–50.