

## DESCRIPTION OF PSEUDOCHARACTERS' SPACE ON FREE PRODUCT OF GROUPS

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*Abstract.* Let  $G = A * B$  be a free product of groups  $A$  and  $B$ . A description is given of the space of real-valued functions  $\varphi$  on the group  $G$  satisfying the following conditions:

- 1) the set  $\{\varphi(xy) - \varphi(x) - \varphi(y); x, y \in G\}$  is bounded;
- 2)  $\varphi(x^n) = n\varphi(x)$  for any  $x \in G$  and any  $n \in \mathbb{Z}$

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