

L^p INEQUALITIES FOR POLAR DERIVATIVES OF POLYNOMIALS

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Abstract. Let $p(z) = \sum_{v=0}^n a_v z^v$ be a polynomial of degree n and for $\alpha \in \mathcal{C}$, let $D_\alpha p(z) = np(z) + (\alpha - z)p'(z)$ denote the polar derivative of the polynomial $p(z)$ with respect to α . It is well known that the polar derivative generalizes the ordinary derivative. In this paper, we obtain *L^p* inequalities for polar derivatives of polynomials satisfying $p(z) \equiv z^n p(\frac{1}{z})$ and for polynomials satisfying $p(z) \equiv z^n p(\frac{1}{z})$. Our results generalize several results in this direction.

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