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NORM INEQUALITIES FOR SOME SINGULAR INTEGRAL OPERATORS

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Abstract. Let $\mathscr{B}$ be a von Neumann algebra and P a selfdjoint projection. For A and B in $\mathscr{B}$, set $S_{A,B} = AP + BQ$ where $Q = I - P$. The operator $S_{A,B}$ will be called a singular integral operator. When $\mathscr{B} = L^{\infty}(T)$ where $L^{\infty}(T)$ is the usual Lebesgue space on the unit circle and P is an analytic projection, in [6] we established formulae for norms of $S_{A,B}$ and $(S_{A,B})^{-1}$. In this paper, if $\mathscr{A} = \{D \in \mathscr{B} : PDP = DP\}$ and $(\mathscr{B}, \mathscr{A}, P)$ has a lifting property, then we will establish formulae of norms of $S_{A,B}$ and $(S_{A,B})^{-1}$. These formulae are operator theoretic and different from the previous ones. There are several examples such that $(\mathscr{B}, \mathscr{A}, P)$ has a lifting property. As result, we give several interesting inequalities.

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