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ON SOME INTEGRAL INEQUALITIES OF OPIAL TYPE

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Abstract. Integral inequalities of the form

$$\int_I s|h|^p|h'|dt \leqslant \int_I r|h'|^{p+1}dt, \qquad h \in H,$$

are derived, where $p > 0$, $I = (\alpha, \beta)$, $-\infty \leqslant \alpha < \beta \leqslant \infty$, r and s are given real functions of the variable t, H is the class of functions h, which are absolutely continuous on I and satisfy the integral condition $\int_I r|h'|^{p+1}dt < \infty$, as well as one of the following boundary conditions: $h(\alpha) = 0$ or $h(\beta) = 0$.

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