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DUALITY PRINCIPLES AND REDUCTION THEOREMS

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Abstract. We introduce a fairly general class of Banach function spaces Λ_X given by $\|f\|_{\Lambda_X} := \|f_\mu^*\|_X$, where f is defined on a totally σ–finite non-atomic measure space $(\mathcal{R}, \mu)$, f_μ^* is the non-increasing rearrangement of f with respect to μ and X is certain rearrangement-invariant space over the interval $(0, \mu(\mathcal{R}))$. This class contains for example classical Lorentz spaces. We prove a general duality principle for these spaces and present several applications. In particular, we prove theorems which enable us to reduce weighted inequalities involving integral operators restricted to monotone functions to certain more manageable weighted inequalities. Reduction theorems are then applied to obtain a characterization of embeddings between Λ_X spaces.

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