

## INEQUALITIES FOR POSITIVE LINEAR MAPS ON HERMITIAN MATRICES

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*Abstract.* The aim of this work is to generalize the main inequalities in [9] as follows: Let  $A$  be a Hermitian matrix, let  $\Phi$  be a normalized positive linear map, let  $f$  and  $g$  be real valued continuous functions and let  $F(u, v)$  be a real valued function matrix non-decreasing in its first variable. Real constants  $\alpha$  and  $\beta$  such that

$$\alpha I \leq F[\Phi(f(A)), g(\Phi(A))] \leq \beta I$$

are determined. If  $f$  is a concave (resp. convex) function then the determination of  $\beta$  (resp.  $\alpha$ ) is reduced to solving a single variable maximization (resp. minimization) problem. Some applications of these results to the power function, the means and the Hadamard product are also given.

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