

STABILITY OF SEMILINEAR STOCHASTIC EVOLUTION EQUATIONS WITH MONOTONE NONLINEARITY

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Abstract. In this paper, we consider the exponentially asymptotic stability of the mild solutions of semilinear stochastic evolution equations of monotone type. An Itô-type inequality is our main tool to study the stability in the p -th moment and almost sure sample-path stability of the mild solutions. We also give some examples to illustrate the applications of the theorems.

Mathematics subject classification (1991): 60H15, 34G20.

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