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MINMAX PROBLEMS FOR FRACTIONAL PARTS OF REAL NUMBERS

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Abstract. The view-obstruction problem for the n-dimensional cube with side 1 can be interpreted as the problem of evaluating the function $\kappa(n) = \inf \max_{0\leqslant x\leqslant 1} \min_{1\leqslant k\leqslant n} \| v_k x \|$, where the infimum is taken over all n-tuples $v_1, \cdots, v_n$ of positive integers. So the following could perhaps be called "generalized view-obstruction problems": given a periodic function ϕ, an interval I and a set of integers $\mathscr{S}$, find

$$(i)\ \ \min_{x\in I}\max_{s\in\mathscr{S}} \phi(sx), \qquad (ii)\ \ \max_{x\in I}\min_{s\in\mathscr{S}} \phi(sx).$$

We study minmax problems of this nature where

$$\phi(x) = \{x\}^{\alpha}(1-\{x\}) \text{ and } \left\{(x-1/2)^{\alpha}\right\},$$

and

$$I = [0,1],\ \ \mathscr{S} = \{1,\cdots,N\}.$$

Here $\{x\}$ denotes the fractional part of x, and $N \geqslant 2$ and $\alpha \geqslant 1$ are integers.

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