MINMAX PROBLEMS FOR FRACTIONAL PARTS OF REAL NUMBERS

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Abstract. The view-obstruction problem for the $n$-dimensional cube with side 1 can be interpreted as the problem of evaluating the function $\kappa(n) = \inf \max_{0 \leq x \leq 1} \min_{1 \leq k \leq n} \| v_k x \|$, where the infimum is taken over all $n$-tuples $v_1, \cdots, v_n$ of positive integers. So the following could perhaps be called “generalized view-obstruction problems”: given a periodic function $\phi$, an interval $I$ and a set of integers $\mathcal{S}$, find

\begin{align*}
(i) & \quad \min_{x \in I} \max_{s \in \mathcal{S}} \phi(sx), \\
(ii) & \quad \max_{x \in I} \min_{s \in \mathcal{S}} \phi(sx).
\end{align*}

We study minmax problems of this nature where

$$\phi(x) = \{x\}^\alpha (1 - \{x\}) \text{ and } \{(x - 1/2)^\alpha\},$$

and

$$I = [0, 1], \; \mathcal{S} = \{1, \cdots, N\}.$$

Here $\{x\}$ denotes the fractional part of $x$, and $N \geq 2$ and $\alpha \geq 1$ are integers.

Key words and phrases: View-obstruction problem, minmax, fractional parts.

REFERENCES