RELATIVE BOUNDEDNESS–COMPACTNESS INEQUALITIES
FOR A SECOND ORDER DIFFERENTIAL OPERATOR

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Abstract. For a one-term second order differential operator with power coefficients and in the limit circle case, we give necessary and sufficient conditions for perturbations to be relatively bounded or relatively compact. These conditions are expressed in terms of integrals of the coefficients of the perturbing operators and are easily verified in many cases. An application is given to the energy operator of the hydrogen atom.

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