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COMPACTNESS OF THE EMBEDDING OPERATORS FOR ROUGH DOMAINS

VLADIMIR GOL'DSHTEIN AND ALEXANDER G. RAMM

Abstract. New classes of non-smooth bounded domains D, for which the embedding operator from $H^1(D)$ into $L^2(D)$ is compact, are introduced. These classes include, in particular, the domains whose boundary locally are graphs of $C-$ functions, but also contain much larger classes of domains. Examples of non-smooth domains for which the above embedding is compact are given. Applications to scattering by rough obstacles are mentioned.

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