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ESTIMATIONS IN HÖLDER'S TYPE INEQUALITIES

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Abstract. Using a technique due to Ozeki, we give an upper bound of

$$\left(\sum a_k^p\right)^{1/p}\left(\sum b_k^q\right)^{1/q} - \lambda \sum a_k b_k$$

for $\lambda > 0$, for $p > 1$, $q > 1$ satisfying $1/p + 1/q = 1$, and for n-tuples $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ of positive numbers under certain conditions. This yields a complement of Hölder's inequality. The estimation with a parameter λ enables us to unify the discussions on difference and ratio inequalities derived from Hölder's inequality.

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