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INEQUALITIES FOR GENERALIZED WEIGHTED MEAN VALUES OF CONVEX FUNCTION

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Abstract. In the article, using the Tchebycheff's integral inequality, the suitable properties of double integral and the Cauchy's mean value theorem in integral form, the following result is proved:

Suppose $f(x)$ is a positive differentiable function and $w(x) \not\equiv 0$ an integrable nonnegative weight on the interval $[a,b]$, if $f'(x)$ and $f'(x)/w(x)$ are integrable and both increasing or both decreasing, then, for all real numbers r and s, we have

$$M_{w,f}(r,s;a,b) < E\Big(r+1,s+1;f(a),f(b)\Big); \qquad (*)$$

if one of the functions $f'(x)$ or $f'(x)/w(x)$ is nondecreasing and the other nonincreasing, then inequality $(*)$ reverses. Where $E(r,s;a,b)$ and $M_{w,f}(r,s;a,b)$ denote the extended mean values and the generalized weighted mean values of function f with two parameters r, s and weight w, respectively.

This inequality generalizes the Hermite-Hadamard's inequality, and the like.

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