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EQUIVALENCE OF THE HÖLDER–ROGERS AND MINKOWSKI INEQUALITIES

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Abstract. It is well-known that the Hölder-Rogers inequality implies the Minkowski inequality. Infantozzi [6] observed implicitely and Royden [15] proved explicitely that the reverse implication is also true. In this note we discuss and give a new proof of this perhaps surprising fact.

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