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AN EXTENSION OF THE SIDON–FOMIN TYPE INEQUALITY AND ITS APPLICATIONS

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Abstract. An extension of the Sidon-Fomin type inequality [5] is made by considering the r-th derivate of the Dirichlet's kernel $D_k^{(r)}$ instead of D_k. Namely, two different proofs of the following inequality

$$\int_0^{\pi} \left| \sum_{k=0}^{n} \alpha_k D_k^{(r)}(x) \right| dx = O\left((n+1)^{r+1}\right), \quad |\alpha_k| \leqslant 1 \quad \text{for all} \quad k \qquad (*)$$

are given. Applying the inequality $(*)$ it's shown that the new class S_r is a subclass of $BV \cap C_r$, $r = 0, 1, 2, \ldots$ where C_r is the extension of the Garret-Stanojević class [7] and BV is the class of null sequences of bounded variation. Also, in this paper an extension of the theorem for convergence and integrability for cosine series of a [6] is made by considering the class S_r, $r = 0, 1, 2, \ldots$ instead of S.

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