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SOME GENERALIZATIONS FOR A THEOREM BY LANDAU

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Abstract. Let $\pi(x)$ be the number of primes not exceeding x. E. Landau made the following conjecture: $\pi(2x) \leqslant 2\pi(x)$ for integer $x \geqslant 2$. In 1966 Rosser and Schoenfeld proved this conjecture. In the present paper we establish upper bounds for $\pi(x+y)$. Taking the particular case $x = y$, we find again Landau's inequality.

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