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# ON A CLASS OF MEANS OF SEVERAL VARIABLES

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*Abstract.* The aim of this paper is to solve the comparison and equality problems of $L$–conjugate means of $n \geqslant 2$ variables defined by

$$L_\varphi^*(x_1, x_2, \ldots, x_n) := \varphi^{-1}\left(\frac{\varphi(x_1) + \varphi(x_2) + \cdots + \varphi(x_n) - \varphi(L(x_1, x_2, \ldots, x_n))}{n-1}\right),$$

where $L : I^n \to I$ is a symmetric mean on the open real interval $I$ and $\varphi : I \to \mathbb{R}$ is continuous strictly monotonic function. The homogeneous $L$–conjugate means are also described. In the last section, the arithmetic mean is characterized as being the only mean that is conjugate arithmetic and quasiarithmetic.

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