# LOWER AND UPPER SOLUTIONS FOR SINGULAR DERIVATIVE DEPENDENT DIRICHLET PROBLEM

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*Abstract.* In this work, we consider the Dirichlet problem

$$u'' + f(t, u, u') = 0,$$
$$u(a) = 0, \quad u(b) = 0,$$

with $f$ singular at $t = a$, $t = b$ and for $u = 0$ and extend previous results concerning the case $f$ independent of $u'$. To this aim we extend the lower and upper solution method in order to work with solutions in $W^{1,1}(a, b) \cap W^{2,1}_{loc}(a, b)$ as well as with lower and upper solutions having unbounded derivatives.

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