ON EMBEDDINGS BETWEEN CLASSICAL LORENTZ SPACES

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Abstract. Let \( p \in (0, \infty) \) and \( v \) be a weight on \((0, \infty)\) and let \( \Lambda^p(v) \) be the classical Lorentz space, determined by the norm \( \|f\|_{\Lambda^p(v)} := \left( \int_0^\infty (f^*(t))^p v(t) \, dt \right)^{1/p} \). When \( p \in (1, \infty) \), this space is known to be a Banach space if and only if \( v \) is non-increasing, while it is only equivalent to a Banach space if and only if \( \Lambda^p(v) = \Gamma^p(v) \), where \( \|f\|_{\Gamma^p(v)} := \left( \int_0^\infty (f^{**}(t))^p v(t) \, dt \right)^{1/p} \). We may thus conclude that, for \( p \in (1, \infty) \), the space \( \Lambda^p(v) \) is equivalent to a Banach space if and only if the norm of a function \( f \) in it can be expressed in terms of \( f^{**} \). We study the question whether an analogous assertion holds when \( p = 1 \). Motivated by this problem, we consider general embeddings between four types of classical and weak Lorentz spaces, namely, \( \Lambda^p(v) \), \( \Lambda^{p,\infty}(v) \), \( \Gamma^p(v) \), \( \Gamma^{p,\infty}(v) \), where \( \Lambda^{p,\infty}(v) \) and \( \Gamma^{p,\infty}(v) \) are certain weak analogues of the spaces \( \Lambda^p(v) \) and \( \Gamma^p(v) \), respectively. We present a unified approach to these embeddings, based on rearrangement techniques. We survey all the known results and prove new ones. Our main results concern the embedding \( \Gamma^{p,\infty}(v) \hookrightarrow \Lambda^q(w) \) which had not been characterized so far. We apply our results to the characterization of associate spaces of classical and weak Lorentz spaces and we give a characterization of fundamental functions for which the endpoint Lorentz space and the endpoint Marcinkiewicz space coincide.


Key words and phrases: Classical Lorentz spaces, continuous embeddings, weighted inequalities for non-increasing functions.

REFERENCES


