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# A SYSTEM OF GENERALIZED AUXILIARY PROBLEMS PRINCIPLE AND A SYSTEM OF VARIATIONAL INEQUALITIES

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*Abstract.* The approximation-solvability of a system of nonlinear variational and quasivariational inequalities $(SNVQVI)$

$$\langle F_1(x^*,y^*), x-x^* \rangle \geqslant 0 \quad \text{for all} \quad x \in X,$$

and

$$\langle F_2(x^*,y^*), g(y)-g(y^*) \rangle \geqslant 0 \quad \text{for all} \quad g(y) \in Y,$$

where $X$ and $Y$, respectively, are nonempty closed convex subsets of $\mathbf{R}^m$ and $\mathbf{R}^n$ and related $F_1 : X \times Y \to \mathbf{R}^m$ and $F_2 : X \times Y \to \mathbf{R}^n$ are any mappings such that $F = (F_1, F_2)$ is $g$-$\gamma$-partially relaxed monotone, is presented. Here $g : \mathbf{R}^n \to \mathbf{R}^n$ is any mapping.

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## REFERENCES

[1] C. BAIOCCHI AND A. CAPELO, *Variational and Quasivariational Inequalities*, Wiley & Sons, New York, 1984.
[2] G. COHEN, *Auxiliary problem principle extended to variational inequalities*, J. Optim. Theo. Appl. **59** (2) (1988), 325–333.t.
[3] J. C. DUNN, *Convexity, monotonicity and gradient processes in Hilbert spaces*, J. Math. Anal. Appl. **53** (1976), 145–158.
[4] J. S. GUO AND J. C. YAO, *Extension of strongly nonlinear quasivariational inequalities*, Appl. Math. Letters **5** (3) (1992), 35–38.
[5] B. S. HE, *A new method for a class of linear variational inequalities*, Math. Programming **66** (1994), 137–144.
[6] B. S. HE, *A class of projection and contraction methods for monotone variational inequalities*, Applied Math. Optim. **35** (1997), 69–76.
[7] D. KINDERLEHRER AND G. STAMPACCHIA, *An Introduction to Variational Inequalities*, Academic Press, New York, 1980.
[8] G. M. KORPELEVICH, *The extragradient method for finding saddle points and other problems*, Matecon **12** (1976), 747–756.
[9] P. MARCOTTE AND J. H. WU, *On the convergence of projection methods*, J. Optim. Theory Appl. **85** (1995), 347–362.
[10] J. S. PANG AND D. CHAN, *Iterative methods for variational and complementarity problems*, Math. Programming **24** (1982), 284–313.
[11] R. U. VERMA, *Nonlinear variational and constrained hemivariational inequalities involving relaxed operators*, ZAMM **77** (5) (1997), 387–391.
[12] R. U. VERMA, *RKKM mapping theorems and variational inequalities*, Math. Proc. Royal Irish Acad. **98A** (2) (1998), 131–138.

[13] R. U. VERMA, *Generalized pseudocontractions and nonlinear variational inequalities*, Publ. Debrecen **33** (1-2) (1998), 23–28.
[14] R. U. VERMA, *An iterative algorithm for a class of nonlinear variational inequalities involving generalized pseudocontractions*, Math. Sci. Res. Hot–Line **2** (5) (1998), 17–21.
[15] R. U. VERMA, *Strongly nonlinear quasivariational inequalities*, Math. Sci. Res. Hot–Line **3** (2) (1999), 11–18.
[16] R. U. VERMA, *Approximation–solvability of nonlinear variational inequalities involving partially relaxed monotone (prm) mappings*, Adv. Nonlinear Var. Inequal. **2** (2) (1999), 137–148.
[17] R. U. VERMA, *An extension of a class of nonlinear quasivariational inequality problems based on a projection method*, Math. Sci. Res. Hot–Line **3** (5) (1999), 1–10.
[18] R. U. VERMA, *Approximation–solvability of a system of nonlinear quasivariational inequalities involving partially relaxed monotone mappings*, Math. Sci. Res. Hot–Line **4** (4) (2000), 47–61.
[19] R. U. VERMA, *On a class of nonlinear implicit variational inequalities involving partially relaxed monotone and partially strongly monotone mappings*, Math. Inequal. Appl. (to appear).
[20] R. U. VERMA, *A new class of iterative algorithms for approximation–solvability of nonlinear variational inequalities*, Computers Math. Appl. (to appear).
[21] E. ZEIDLER, *Nonlinear Functional Analysis and its Applications I*, Springer–Verlag, New York, 1986.
[22] D. L. ZHU AND P. MARCOTTE, *Co–coercivity and its role in the convergence of iterative schemes for solving variational inequalities*, SIAM J. Optim. **6** (3) (1996), 714–726.