

## MATRIX INEQUALITIES BY MEANS OF BLOCK MATRICES

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*Abstract.* We first show a weak log-majorization inequality of singular values for partitioned positive semidefinite matrices which will imply some existing results of a number of authors, then present some basic matrix inequalities and apply them to obtain a number of matrix inequalities involving sum, ordinary product and Hadamard product.

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### REFERENCES

- [1] A. ALBERT, *Conditions for positive and nonnegative definiteness in terms of pseudoinverses*, SIAM J. Appl. Math., Vol. 17, No. 2, March 1969.
- [2] P. A. BEKKER, *The positive semidefiniteness of partitioned matrices*, Linear Algebra and Its Applications 111:261-278 (1988).
- [3] R. BHATIA, *Matrix Analysis*, Springer-Verlag, New York, 1997.
- [4] R. BHATIA AND F. KITANEH, *Norm Inequalities for Positive Operators*, Letters in Math Phys. 43: 225–231 (1998).
- [5] C. H. FITZGERALD AND R. A. HORN, *On fractional Hadamard powers of positive definite matrices*, J. of Math Analysis and Appl. 61, pp. 633–642 (1977).
- [6] P. R. HALMOS, *Linear Algebra Problem Book*, Mathematical Association of America, Washington, DC, 1995.
- [7] R. A. HORN AND C. R. JOHNSON, *Topics in Matrix Analysis*, Cambridge University Press, New York, 1991.
- [8] C. R. JOHNSON, *Partitioned and Hadamard product matrix inequalities*, J. Res. Nat. Bur. Standards, 83: 585–591 (1978).
- [9] V. V. PRASOLOV, *Problems and Theorems in Linear Algebra*, American Mathematical Society, Providence, RI, 1994.
- [10] G. VISICK, *A quantitative version of the observation that the Hadamard product is a principal submatrix of the Kronecker product*, Linear Algebra and Its Applications 304 (2000) 45-68.
- [11] G. VISICK, *An algebra relationship between the Hadamard and Kronecker product with some applications*, Bull. Soc. Math. Belg. 42(1990), 3, Ser. B, pp. 275–283.
- [12] B.-Y. WANG, B.-Y. XI AND F. ZHANG, *Some inequalities for sum and product of positive semidefinite matrices*, Linear Algebra and Its Applications, 293 (1999) 39–49.
- [13] X.-Z. ZHAN, *Singular values of differences of positive semidefinite matrices*, SIAM Matrix Analysis and Appl. Vol. 22, No. 3 (2000), 819–823.
- [14] F. ZHANG, *Another Proof of a Singular Value Inequality Concerning Hadamard Products of Matrices*, Linear and Multilinear Algebra, Vol. 22 (1988) 307–311.
- [15] F. ZHANG, *Notes on Hadamard products of matrices*, Linear and Multilinear Algebra, Vol. 25 (1989) 237–242.
- [16] F. ZHANG, *Schur complements and matrix inequalities in the Löwner ordering*, Linear Algebra and Its Applications 321 (2000) 399–410.
- [17] F. ZHANG, *Matrix Theory: Basic Results and Techniques*, Springer-Verlag, New York, 1999.