

INTEGRAL MEAN ESTIMATES FOR POLYNOMIALS WITH RESTRICTED ZEROS

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Abstract. Let $P(z)$ be a polynomial of degree n which does not vanish in the disk $|z| < K$. For $K = 1$, it is known that for $0 < q < \infty$,

$$\left\{ \frac{1}{2\pi} \int_0^{2\pi} |P(Re^{i\theta})|^q d\theta \right\}^{1/q} \leq B_q \left\{ \frac{1}{2\pi} \int_0^{2\pi} |P(e^{i\theta})|^q d\theta \right\}^{1/q},$$

where

$$B_q = \left\{ \frac{1}{2\pi} \int_0^{2\pi} |1 + R^n e^{in\alpha}|^q d\alpha \right\}^{1/q} / \left\{ \frac{1}{2\pi} \int_0^{2\pi} |1 + e^{in\alpha}|^q d\alpha \right\}^{1/q}.$$

In this paper we present a generalization of this result by considering the case $K \geq 1$. We shall also prove a similar result for polynomials having all their zeros in $|z| \leq K$, where $K \geq 1$.

Mathematics subject classification (2000): 26D05, 30D15, 41A17.

Key words and phrases: Integral mean estimates, admissible operator, complex domain inequalities.

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