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THE PROPERTIES OF FOUR ELEMENTS IN ORLICZ–MUSIELAK SPACES

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Abstract. "The property of four elements" (PFE), closely related to the isotonicity of the metric projection operator, has been recently introduced and proved in ordered Hilbert spaces, L_p-spaces and Orlicz-Musielak spaces (see [5], [6], [12]). Moreover, a dual inequality named "the upper property of four elements" $(UPFE)$ for norms in L_p-spaces has been discussed in [13].

In this paper we prove that the inequality $(UPFE)$ holds in all Orlicz-Musielak spaces generated by a convex modular. It is also shown that both properties of four elements can be reversed (with some other constants) if the function generating the modular satisfies the condition (Δ_2). This generalizes Theorems 3.1, 3.4 from [13].

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