

SPECTRAL ORDER $A \succ B$ IF AND ONLY IF $A^{2p-r} \geq (A^{\frac{-r}{2}} B^p A^{\frac{-r}{2}})^{\frac{2p-r}{p-r}}$ FOR ALL $p > r \geq 0$ AND ITS APPLICATION

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Abstract. New characterization of spectral order $A \succ B$ is given as follows:

$$A \succ B \text{ if and only if } A^{2p-r} \geq (A^{\frac{-r}{2}} B^p A^{\frac{-r}{2}})^{\frac{2p-r}{p-r}} \text{ for all } p > r \geq 0$$

and also its application is given.

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