SPECTRAL ORDER $A \succ B$ IF AND ONLY IF $A^{2p-r} \succeq (A^{-\frac{r}{2}}B^{p}A^{-\frac{r}{2}})^{\frac{2p-r}{p-r}}$

FOR ALL $p > r \geq 0$ AND ITS APPLICATION

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Abstract. New characterization of spectral order $A \succ B$ is given as follows:

$A \succ B$ if and only if $A^{2p-r} \succeq (A^{-\frac{r}{2}}B^{p}A^{-\frac{r}{2}})^{\frac{2p-r}{p-r}}$ for all $p > r \geq 0$

and also its application is given.


Key words and phrases: Spectral order, Löwner-Heinz inequality, generalized Furuta inequality.

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