

## SPHERICALLY SYMMETRIC FUNCTIONS WITH A CONVEX SECOND DERIVATIVE AND APPLICATIONS TO EXTREMAL PROBABILISTIC PROBLEMS

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*Abstract.* We describe the class of all functions  $\varphi: [0, \infty) \rightarrow \mathbb{R}$  for which the second derivative  $g''_{\varphi}(x; y, y)$  of the spherically symmetric function  $g_{\varphi}(x) := \varphi(|x|)$  in the direction of  $y$  is convex in  $x$ , where  $x$  and  $y$  are vectors in a Hilbert space  $H$  and  $|\cdot|$  is the norm in  $H$ . Applications to extremal probabilistic problems are given.

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*Key words and phrases:* Spherically symmetric (spherically invariant, rotation-invariant, elliptically contoured) functions; convexity; convex second derivative; extremal probabilistic problems; probabilistic inequalities; Khinchine inequality; generalized moments; generalized moment comparison inequalities; random multilinear (multi-affine) forms; Rademacher chaos; Hilbert space; sums of independent random variables; differential inequalities.

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