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GENERAL AUXILIARY PROBLEM PRINCIPLE AND SOLVABILITY OF A CLASS OF NONLINEAR MIXED VARIATIONAL INEQUALITIES INVOLVING PARTIALLY RELAXED MONOTONE MAPPINGS

THEMISTOCLES M. RASSIAS AND RAM U. VERMA

Abstract. The approximation–solvability of the following class of nonlinear variational inequality (NVI) problems based on a new general auxiliary problem principle is presented: Find an element $x^* \in K$ such that

$$\langle T(x^*), x - x^* \rangle + f(x) - f(x^*) \geqslant 0 \quad \text{for all} \quad x \in K,$$

where $T : K \to H$ is a partially relaxed monotone mapping from a nonempty closed convex subset K of a real Hilbert space H into H, and $f : K \to \mathbf{R}$ is a continuous convex function on K. The general auxiliary problem principle is described as follows: for given iterate $x^k \in K$ and for a constant $\rho > 0$, determine x^{k+1} such that (for $k \geqslant 0$)

$$\langle \rho T(x^k) + \rho L(x^{k+1}) + h'(x^{k+1}) - \rho L(x^k) - h'(x^k), x - x^{k+1} \rangle + \rho[f(x) - f(x^{k+1})] \geqslant 0$$

for all $x \in K$, where $L : K \to H$ is any mapping on K, $h : K \to \mathbf{R}$ is a function on K and h' is the derivative of h.

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REFERENCES

[1] C. BAIOCCHI AND A. CAPELO, *Variational and Quasivariational Inequalities*, Wiley & Sons, New York, 1984.
[2] G. COHEN, *Auxiliary problem principle and decomposition of optimization problems*, J. Optim. Theo. Appl. **32** (3) (1980), 277–305.
[3] G. COHEN, *Auxiliary problem principle extended to variational inequalities*, J. Optim. Theo. Appl. **59** (2) (1988), 325–333.t.
[4] J. C. DUNN, *Convexity, monotonicity and gradient processes in Hilbert spaces*, J. Math. Anal. Appl. **53** (1976), 145–158.
[5] J. S. GUO AND J. C. YAO, *Extension of strongly nonlinear quasivariational inequalities*, Appl. Math. Letters **5** (3) (1992), 35–38.
[6] B. S. HE, *A projection and contraction method for a class of linear complementarity problems and its applications,*, Applied Math. Optim **25** (1992), 247–262.
[7] B. S. HE, *A new method for a class of linear variational inequalities*, Math. Programming **66** (1994), 137–144.
[8] B. S. HE, *Solving a class of linear projection equations*, Numer. Math **68** (1994), 71–80.
[9] B. S. HE, *A class of projection and contraction methods for monotone variational inequalities*, Applied Math. Optim. **35** (1997), 69–76.

[10] D. KINDERLEHRER AND G. STAMPACCHIA, *An Introduction to Variational Inequalities*, Academic Press, New York, 1980.
[11] G. M. KORPELEVICH, *The extragradient method for finding saddle points and other problems*, Matecon **12** (1976), 747–756.
[12] P. MARCOTTE AND J. H. WU, *On the convergence of projection methods*, J. Optim. Theory Appl. **85** (1995), 347–362.
[13] J. S. PANG AND D. CHAN, *Iterative methods for variational and complementarity problems*, Math. Programming **24** (1982), 284–313.
[14] R. U. VERMA, *Nonlinear variational and constrained hemivariational inequalities involving relaxed operators*, ZAMM **77** (5) (1997), 387–391.
[15] R. U. VERMA, *RKKM mapping theorems and variational inequalities*, Math. Proc. Royal Irish Acad. **98A** (2) (1998), 131–138.
[16] R. U. VERMA, *Generalized pseudocontractions and nonlinear variational inequalities*, Publ. Math. Debrecen **33** (1-2) (1998), 23–28.
[17] R. U. VERMA, *An iterative algorithm for a class of nonlinear variational inequalities involving generalized pseudocontractions*, Math. Sci. Res. Hot–Line **2** (5) (1998), 17–21.
[18] R. U. VERMA, *Strongly nonlinear quasivariational inequalities*, Math. Sci. Res. Hot–Line **3** (2) (1999), 11–18.
[19] R. U. VERMA, *Approximation–solvability of nonlinear variational inequalities involving partially relaxed monotone (prm) mappings*, Adv. Nonlinear Var. Inequal. **2** (2) (1999), 137–148.
[20] R. U. VERMA, *An extension of a class of nonlinear quasivariational inequality problems based on a projection method*, Math. Sci. Res. Hot–Line **3** (5) (1999), 1–10.
[21] R. U. VERMA, *On a class of projection-contraction methods applied to monotone variational inequalities*, Math. Letters (to appear).
[22] R. U. VERMA, *A new class of iterative algorithms for approximation–solvability of nonlinear variational inequalities*, Computers Math. Appl. (to appear).
[23] E. ZEIDLER, *Nonlinear Functional Analysis and its Applications I*, Springer–Verlag, New York, 1986.
[24] D. L. ZHU AND P. MARCOTTE, *Co–coercivity and its role in the convergence of iterative schemes for solving variational inequalities*, SIAM J. Optim. **6** (3) (1996), 714–726.