AVERAGING OPERATORS ON $l^{\{p_n\}}$ AND $L^{p(x)}$

David E. Edmunds and Aleš Nekvinda

Abstract. We consider the generalized Lebesgue space $L^{p(x)}$ and its discrete analogue $l^{\{p_n\}}$, each given the appropriate Luxemburg norm. Let T_k be the averaging operator given by

$$(T_k a)_n = \frac{1}{k}(a_n + a_{n+1} + \cdots + a_{n+k-1}), a = \{a_n\} \in l^{\{p_n\}}.$$

We show that the T_k are uniformly bounded from $l^{\{p_n\}}$ into $l^{\{p_n\}}$ under certain assumptions on p_n and find a counter–example to show that T_k need not be bounded if these assumptions are not satisfied.

Moreover, we construct a bounded Lipschitz function $p(x)$ on $[0, \infty)$ such that the operator T_s given, for each

$$T_s f(x) = \frac{1}{s} \int_0^s f(t) dt$$

is unbounded on $L^{p(x)}$ for all $s > 0$.

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